Proposal for a Tutorial at SPAA 2024

New Algorithmic Techniques for Scheduling via Integer Linear Programming

by Klaus Jansen, Kiel University, Germany.

Abstract. In this tutorial we present new algorithmic methods and results for scheduling problems on parallel machines. During the last years we have worked on the design of efficient exact algorithms and approximation schemes for scheduling problems on identical and uniform machines. The classical approach to obtain an approximation scheme for identical machines is to use an integer linear program (ILP) for the large rounded jobs and to place the remaining small jobs in a greedy fashion on the machines. Unfortunately the number of integer variables in the ILP is exponential large in $1/\epsilon$ and the running time would be double exponential in $1/\epsilon$. How can we obtain more efficient approximation schemes?

Content. In order to obtain faster (implementable) algorithms for these scheduling problems we studied different ILP formulations. A first approach is to bound the support of an optimum solution (i.e. the number of strictly positive variables of the ILP solution) and to guess these variables. This reduces the running time to almost single exponential in $1/\epsilon^2$. A second approach is to study the structure of optimum solutions in more details. Interestingly we can suppose that there is an optimum solution where only very few machines have more than $O(\log(1/\epsilon))$ different job sizes. This helps to speed up the approximation scheme further to almost single exponential in $1/\epsilon$.

Next we developed new parameterized algorithms for ILPs with few constraints and bounds on the coefficients of the variables. In our ILP formulation, the number of constraints is $O(1/\epsilon \log(1/\epsilon))$ and the largest coefficient is at most $1/\epsilon$. Using algorithms by Eisenbrand and Weismantel or another even faster one by Jansen and Rohwedder we can solve these ILPs very efficient. The algorithms are based on the Steinitz Lemma for a collection of vectors that sum up to the zero vector or on discrepancy bounds for the corresponding matrices. Therefore, to get an even faster and implementable algorithm, we modified the rounding of the job sizes to obtain a different ILP formulation, which has a smaller discrepancy bound. We will explain in the tutorial how to use these techniques and describe how our algorithms work.

We believe that these techniques can be used for many other scheduling, load balancing and packing problems. Finally we describe several complexity results for lower bounds on the running time and mention some interesting open questions in this research area. The tutorial is based on the following papers:

1. Klaus Jansen and Lars Rohwedder: On Integer Programming, Discrepancy, and Convolution, Mathematics of Operations Research 48(3): 1481-1495 (2023).
2. Sebastian Berndt, Hauke Brinkop, Klaus Jansen, Matthias Mnich, and Tobias Stamm: New Support Size Bounds for Integer Programming, Applied to Makespan Minimization on Uniformly Related Machines. ISAAC 2023: 13:1-13:18.
3. Sebastian Berndt, Max A. Deppert, Klaus Jansen, and Lars Rohwedder: Load Balancing: The Long Road from Theory to Practice. ALENEX 2022: 104-116.
4. Lin Chen, Klaus Jansen, and Guochuan Zhang: On the Optimality of Exact and Approximation Algorithms for Scheduling Problems. Journal of Computer and System Science 96: 1-32 (2018).
5. Klaus Jansen, Kim-Manuel Klein, and Jos Verschae: Closing the Gap for Makespan Scheduling via Sparsification Techniques. Mathematics of Operations Research 45(4): 1371-1392 (2020).
6. Klaus Jansen: An EPTAS for Scheduling Jobs on Uniform Processors: Using an MILP Relaxation with a Constant Number of Integral Variables. SIAM Journal on Discrete Mathematics 24(2): 457-485 (2010).

Prerequisites. The tutorial assums knowledge on the level of a master's degree in computer science or mathematics and some previous experience with linear optimization and/or approximation algorithms. Most of the concrete previous knowledge we assume will be covered by algorithmic and complexity courses of a bachelor degree.

Biography. Klaus Jansen studied computer science with minor mathematics at the RWTH Aachen and received his PhD and habilitation in mathematics from the University of Trier in 1990 and 1994, respectively. After research positions at the Max-Planck-Institute for Informatics (MPII) in Saarbrücken and Instituto Dalle Molle de Artificial Intelligence (IDSIA) in Lugano, he obtained a professorship at the University of Kiel in Germany. He works in the Computer Science Department and leads the group Algorithms and Complexity. He was coordinator of two EU Project APPOL I + II, member of three other EU Projects ARACNE, CRESCO and AEOLUS and acquired fourteen DFG projects. He was co-founder of two workshops APPROX on Approximation Algorithms for Combinatorial Optimization Problems and WAOA on Approximation and Online Algorithms.

He published around 110 papers in scientific journal and 180 papers in conferences in computer science; mostly on exact and approximation algorithms and complexity results for combinatorial optimization problems. This includes algorithms for many different scheduling problems, the classical bin packing problem, two and three dimensional packing problems and integer linear programming.